Physical models for micro and nanosystems

Chapter 7: Microsystems for biology

Part 1: Fluids

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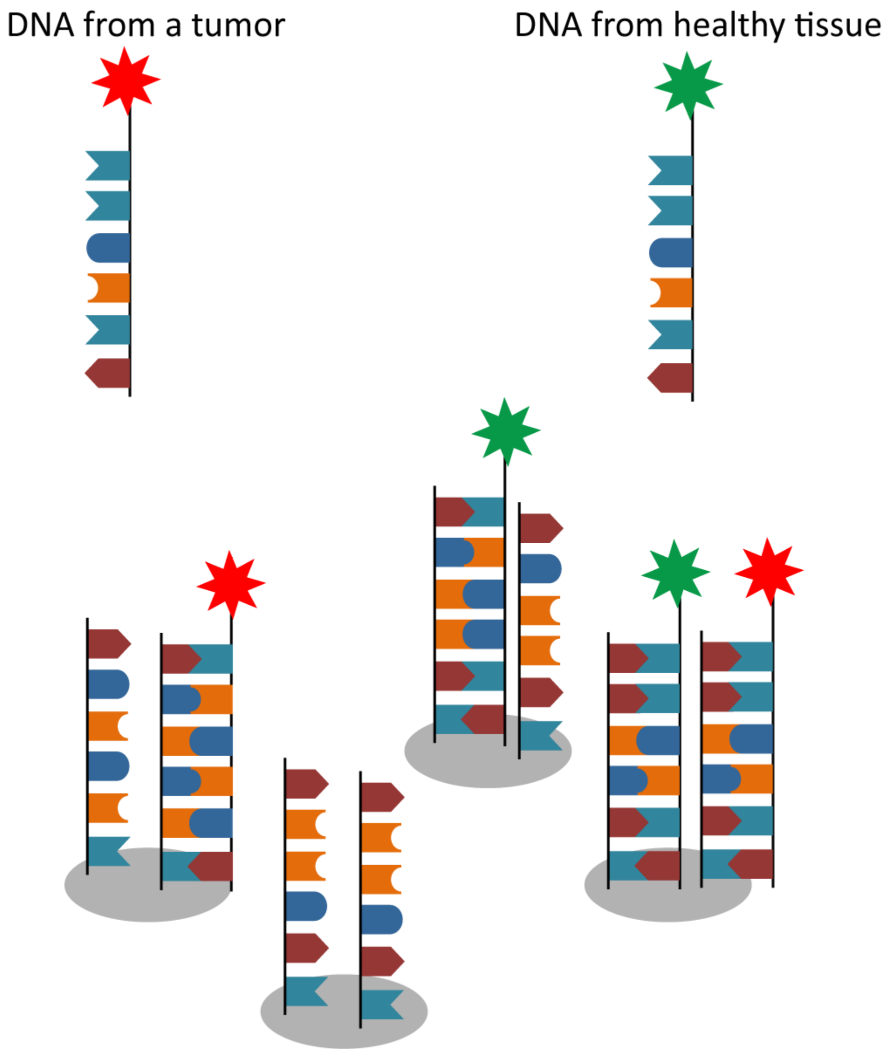
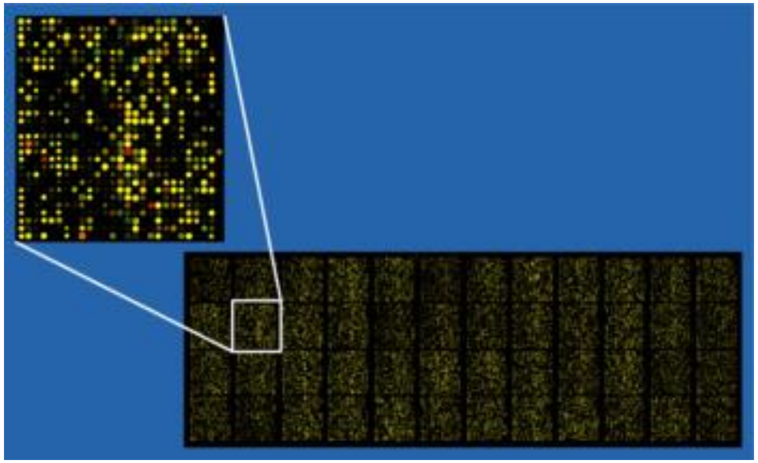
Microsystems for biology

- Integrated systems for performing biological tests
- Example: microarrays

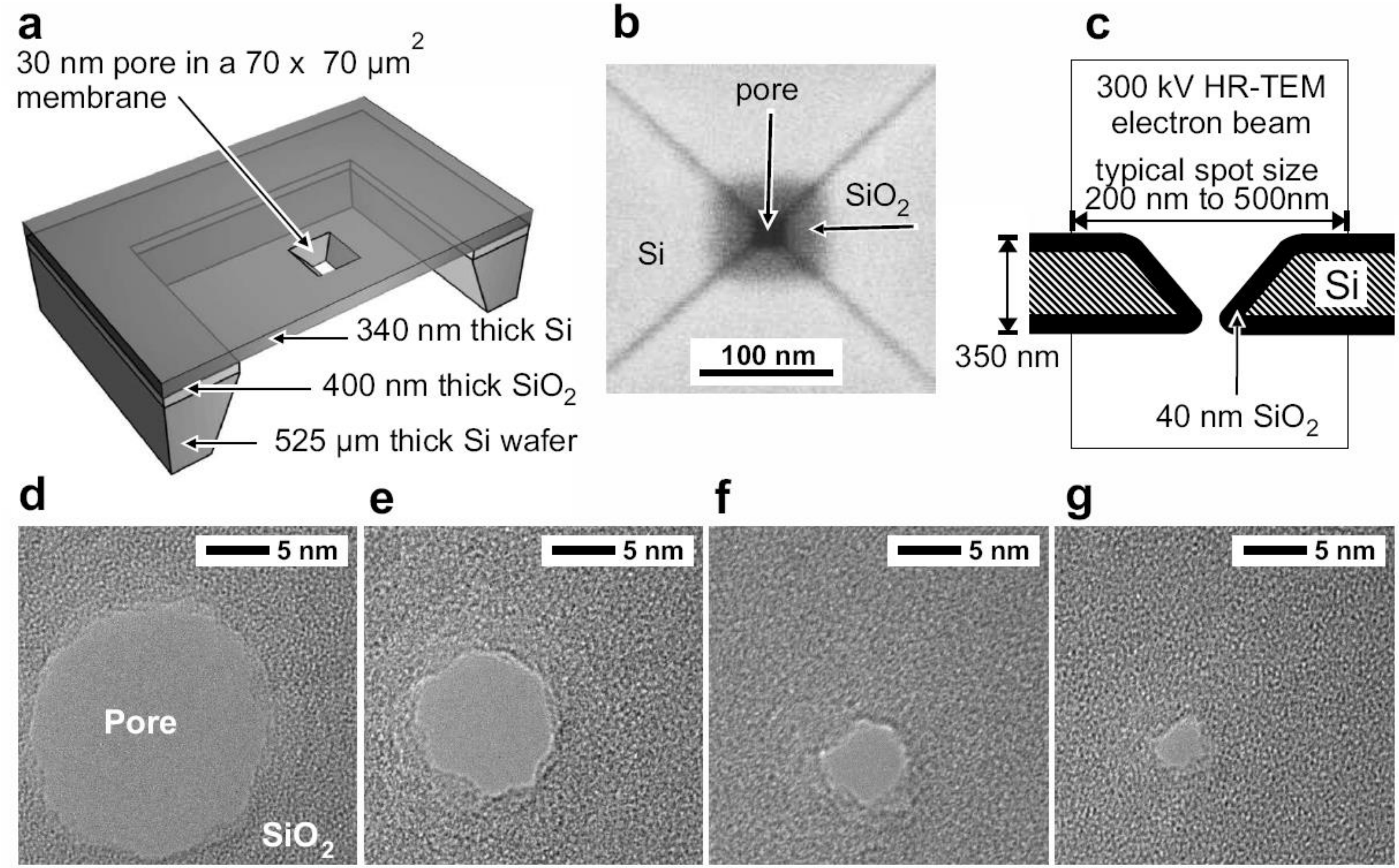




Images from wikipedia

Microsystems for biology

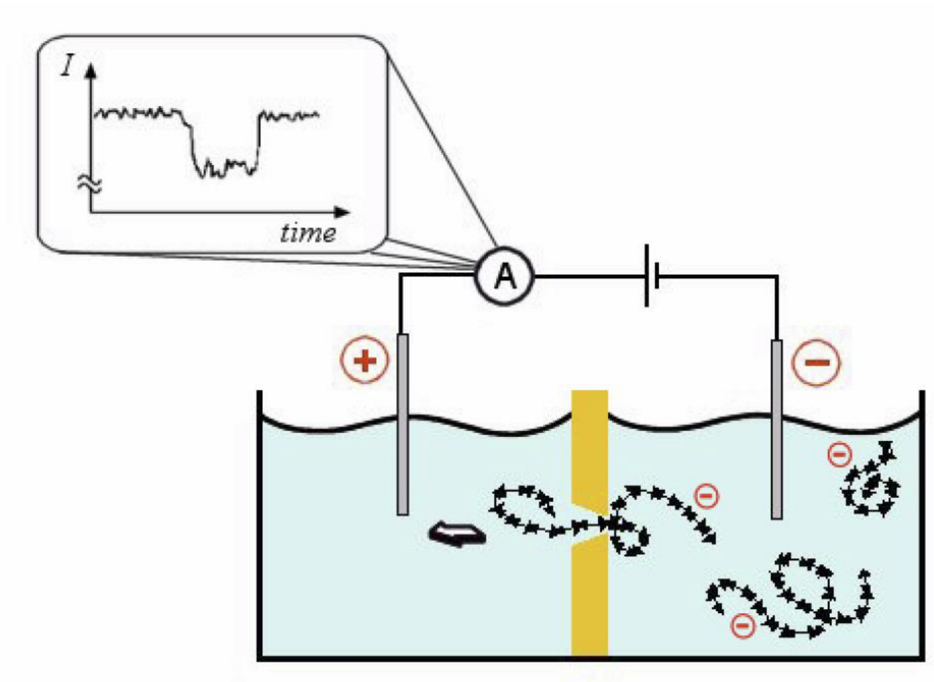
- Integrated systems for performing biological tests
- Example: nanopores



A. J. Storm PhD thesis, TU Delft, 2004

Microsystems for biology

- Integrated systems for performing biological tests
- Example: nanopores



Questions:

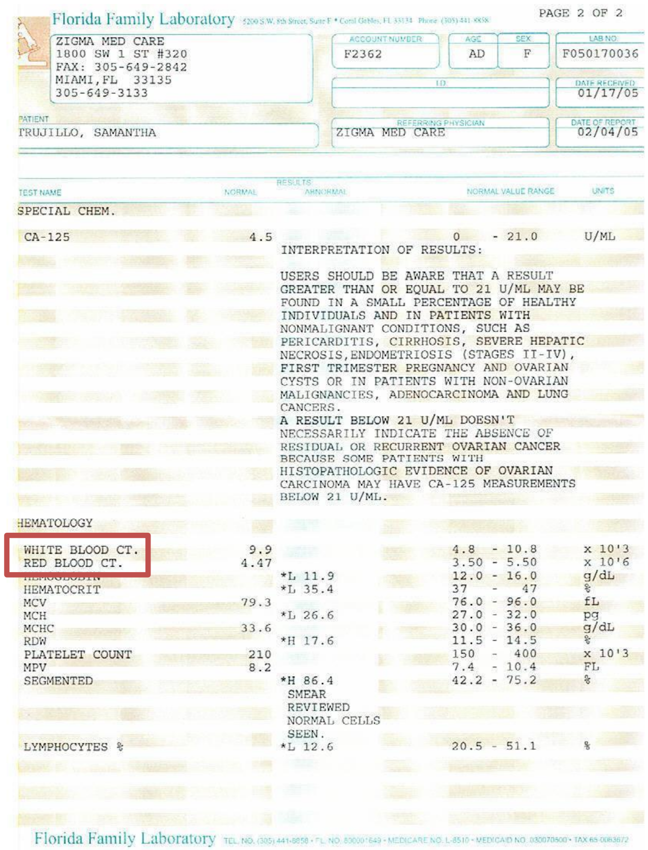
What is the current magnitude?

Electric field around the nanopore?

Counting cells

Coulter counter (Wallace and Joseph Coulter, 1946)

cells passing through a small hole cause a drop in ionic current through the hole



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PAGE 2 OF 2

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ACCOUNT NUMBER: F2362 | AGE: AD | SEX: F | LAB NO: F050170036

I.D.:

DATE RECEIVED: 01/17/05

PATIENT: TRUJILLO, SAMANTHA

REFERRING PHYSICIAN: ZIGMA MED CARE

DATE OF REPORT: 02/04/05

TEST NAME	RESULTS NORMAL	RESULTS ABNORMAL	NORMAL VALUE RANGE	UNITS
SPECIAL CHEM.				
CA-125	4.5		0 - 21.0	U/ML

INTERPRETATION OF RESULTS:

USERS SHOULD BE AWARE THAT A RESULT GREATER THAN OR EQUAL TO 21 U/ML MAY BE FOUND IN A SMALL PERCENTAGE OF HEALTHY INDIVIDUALS AND IN PATIENTS WITH NONMALIGNANT CONDITIONS, SUCH AS PERICARDITIS, CIRRHOSIS, SEVERE HEPATIC NECROSIS,ENDOMETRIOSIS (STAGES II-IV), FIRST TRIMESTER PREGNANCY AND OVARIAN CYSTS OR IN PATIENTS WITH NON-OVARIAN MALIGNANCIES, ADENOCARCINOMA AND LUNG CANCERS.
A RESULT BELOW 21 U/ML DOESN'T NECESSARILY INDICATE THE ABSENCE OF RESIDUAL OR RECURRENT OVARIAN CANCER BECAUSE SOME PATIENTS WITH HISTOPATHOLOGIC EVIDENCE OF OVARIAN CARCINOMA MAY HAVE CA-125 MEASUREMENTS BELOW 21 U/ML.

TEST NAME	RESULTS NORMAL	RESULTS ABNORMAL	NORMAL VALUE RANGE	UNITS
HEMATOLOGY				
WHITE BLOOD CT.	9.9		4.8 - 10.8	x 10'3
RED BLOOD CT.	4.47		3.50 - 5.50	x 10'6
[illegible]		*L 11.9	12.0 - 16.0	g/dL
HEMATOCRIT		*L 35.4	37 - 47	%
MCV	79.3		76.0 - 96.0	fL
MCH		*L 26.6	27.0 - 32.0	pg
MCHC	33.6		30.0 - 36.0	g/dL
RDW		*H 17.6	11.5 - 14.5	%
PLATELET COUNT	210		150 - 400	x 10'3
MPV	8.2		7.4 - 10.4	FL
SEGMENTED		*H 86.4 SMEAR REVIEWED NORMAL CELLS SEEN.	42.2 - 75.2	%
LYMPHOCYTES %		*L 12.6	20.5 - 51.1	%

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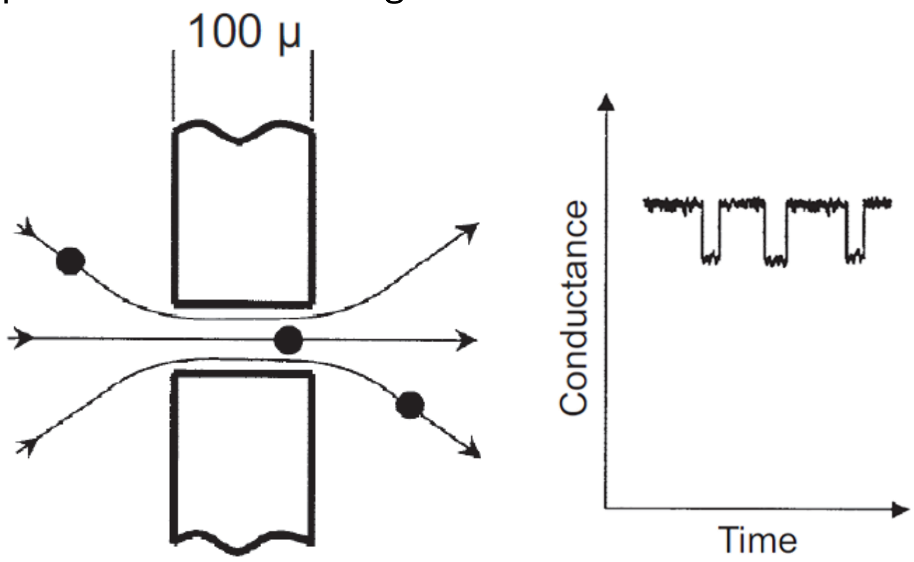
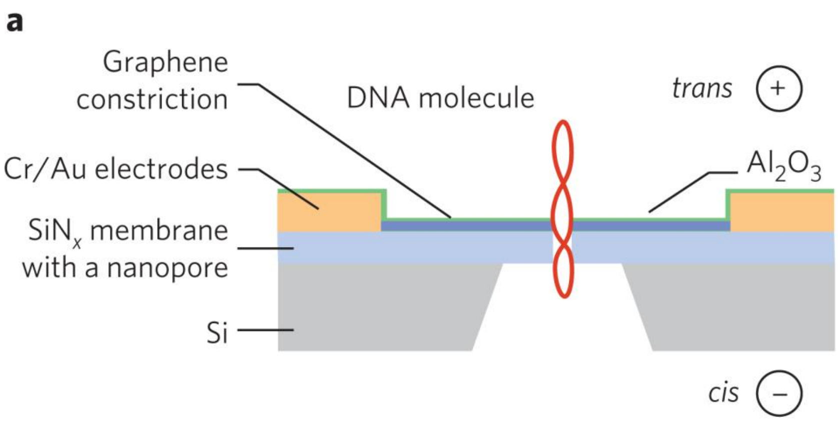
Blood test results

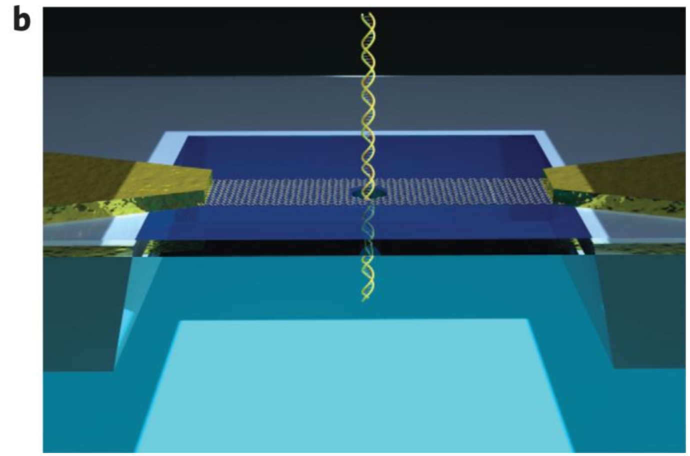


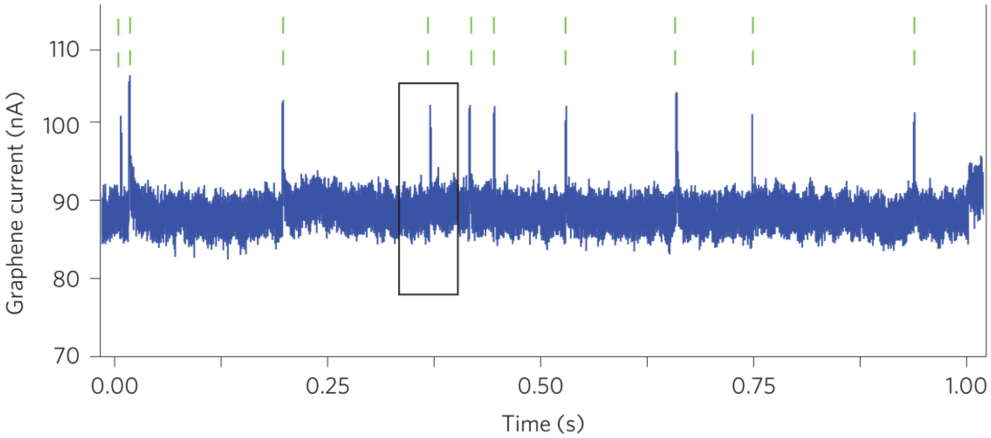
nature nanotechnology
ARTICLES
PUBLISHED ONLINE: 17 NOVEMBER 2013 | DOI: 10.1038/NNANO.2013.240

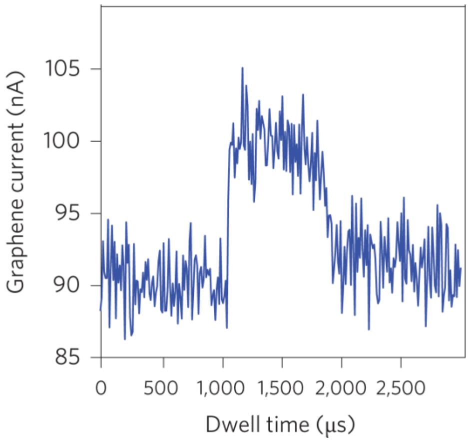
Detecting the translocation of DNA through a nanopore using graphene nanoribbons

F. Traversi[1], C. Raillon[1], S. M. Benameur[2], K. Liu[1], S. Khlybov[1], M. Tosun[2], D. Krasnozhon[2], A. Kis[2] and A. Radenovic[1]*

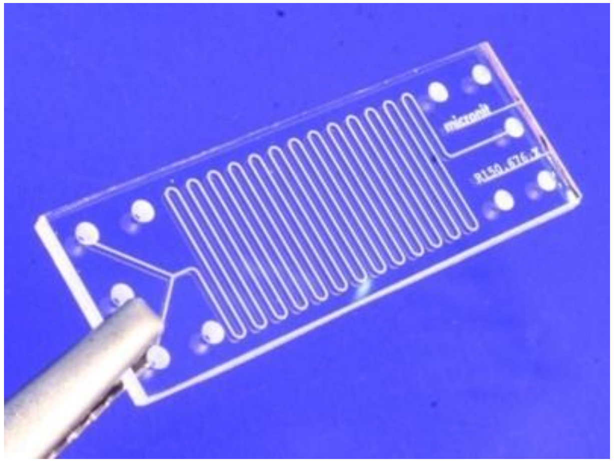


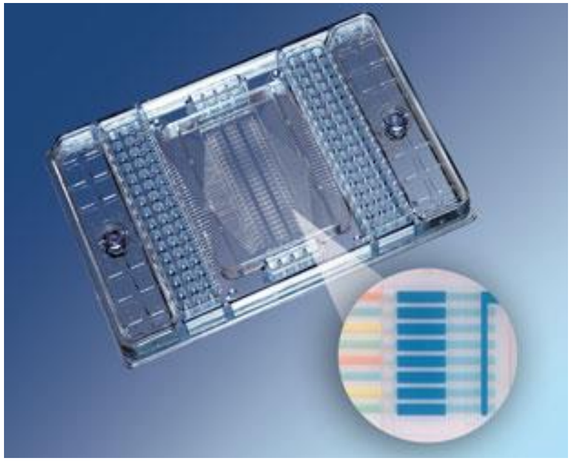


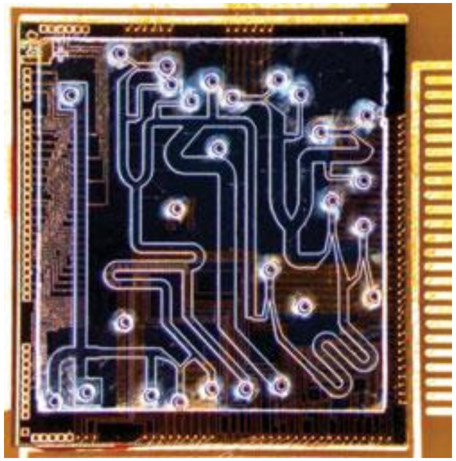




Microfluidics







Topics for this chapter

- Fluid transport
- Heat transport

Characteristics of ideal fluids

- Four assumptions related to ideal fluids:

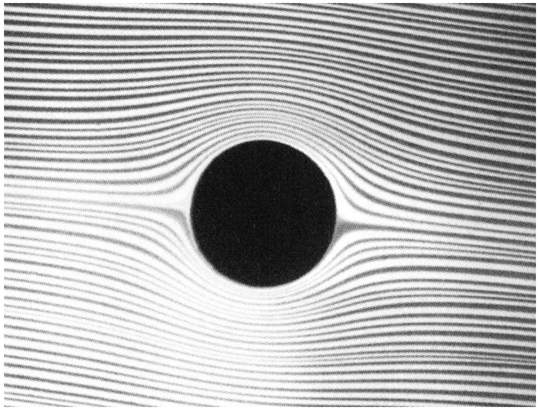
  - **steady flow (laminar flow)** – the velocity of the moving fluid is constant in time (both in magnitude and in direction); the opposite is *turbulent flow*

  - **incompressible flow –** the fluid has a constant and uniform density

  - **nonviscous flow –** objects moving through the fluid do not experience any resistance

  - **irrotational flow** - $\nabla \times \vec{v} = 0$ where $\vec{v}$ is the fluid velocity

Rising smoke



Fluid flowing past a cylinder

Other extreme of viscosity

- University of Queensland Pitch drop experiment – bitumen, 2×10^{11} times more viscous than water

Date	Event	Duration (Months)
1927	Experiment set up	
1930	The stem was cut	
December 1938	1st drop fell	96-107
February 1947	2nd drop fell	99
April 1954	3rd drop fell	86
May 1962	4th drop fell	97
August 1970	5th drop fell	99
April 1979	6th drop fell	104
July 1988	7th drop fell	111
28 November 2000	8th drop fell	148

http://gizmodo.com/a-69-year-old-experiment-finally-worked-for-the-first-t-827373065

http://smp.uq.edu.au/content/pitch-drop-experiment

Continuity equations: Steady flow of an ideal fluid

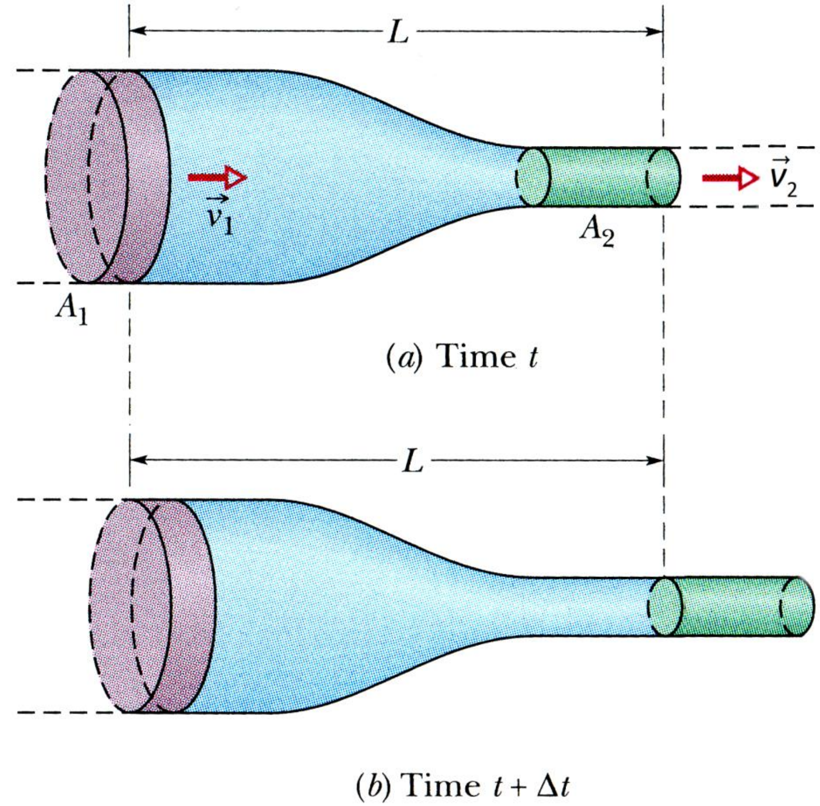
- Let us consider fluid flowing from left to right through a tube segment of length L
- The fluid has speeds v_1 at the left end and v_2 at the right end
- In a time interval Δt a volume ΔV of fluid enters at left end
- Because the fluid is incompressible, the same volume ΔV must emerge from the right segment
- If we consider a tube of uniform cross-sectional area A, the volume ΔV is equal to:

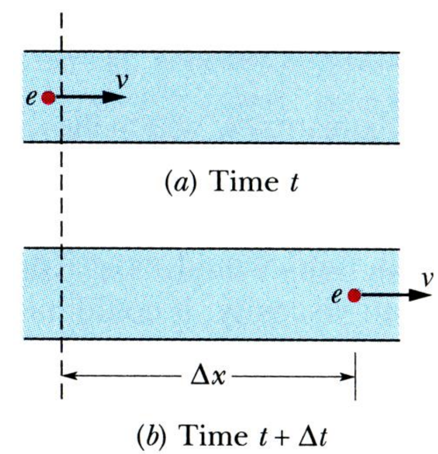
$$\Delta V = A\Delta x = Av\Delta t$$

- Applying this to the tube with varying diameter, we get:

$$\Delta V = A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

$$A_1 v_1 = A_2 v_2 \qquad Av = const.$$ equation of continuity





Fluid dynamics in general

- In the most general case, fluid dynamics is described by a set of differential equations known as Navier – Stokes equations

- These equations are different formulations of the well-known principles of the conservation of mass, momentum and energy

- They are connected through the Reynolds transport theorem which states that the changes of some property L defined over a volume Ω, must be equal to what is lost (or gained) through the boundaries of the volume (surface $\partial\Omega$) plus what is created/consumed by sources and sinks inside the volume Ω.

- This theorem is expressed by the following equation:

$$\frac{d}{dt}\int_{\Omega} L dV = -\int_{\partial\Omega} L\vec{u}\cdot\hat{n}\, dA - \int_{\Omega} Q dV$$

- We can rewrite the second (surface) integral using the Gauss' theorem:

$$\int_{\partial\Omega} L\vec{u}\cdot\hat{n}\, dA = \int_{\Omega} \nabla\cdot(L\vec{u})\, dV$$

where $\vec{u}$ is the flow velocity and Q represents sources and sinks

Fluid dynamics in general

- By combining all the integrals, we get:

$$\int_{\Omega} \left[\frac{\partial L}{\partial t} + \nabla \cdot (L\vec{u}) + Q\right] dV = 0$$

This must be zero for any control volume; this is true only when the integrand is zero, so that:

$$\boxed{\frac{\partial L}{\partial t} + \nabla \cdot (L\vec{u}) + Q = 0}$$

- We can first apply this general equation to mass. In this case $Q = 0$ (there are no sources or sinks for mass) and putting in density for L , we get:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\vec{u}) = 0$$

this is called the mass continuity equation or simply continuity equation

Fluid dynamics in general

- For the case of momentum, things start easily – put momentum written as $\rho\vec{u}$ for L:

$$\frac{\partial}{\partial t}(\rho\vec{u}) + \nabla\cdot(\rho\vec{u}\vec{u}) + Q = f$$

- In this picture, the source or sink of momentum is a body force $Q = f$ (force divided by volume)

  Things however get very complicated because the quantity $\vec{u}\vec{u}$ is a dyad, a rank 2 tensor and to make things even worse, we would now have to calculate its divergence

- The end result of that would be this Navier-Stokes equation:

$$\rho\frac{\partial\vec{u}}{\partial t} + \rho(\vec{u}\cdot\nabla)\vec{u} + \nabla p - \eta\nabla^2\vec{u} - (\lambda+\eta)\nabla(\nabla\cdot\vec{u}) = \vec{f}$$

  where

  p is the pressure in the fluid

  η the dynamic fluid viscosity

  λ is a second viscosity coefficient (related to the compressibility of the fluid)

  This equation is sometimes written with $\nu = \eta/\rho$ (kinematic viscosity)

Incompressible fluids

- In order to solve these equations, simplifications have to be made – first one is to assume that the fluid is **incompressible**
- This means that the density ρ is constant in space as well as in time
- Navier-Stokes equations

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \qquad (1)$$

and

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u} \cdot \nabla)\vec{u} + \nabla p - \eta \nabla^2 \vec{u} - (\lambda + \eta)\nabla(\nabla \cdot \vec{u}) = \vec{f} \qquad (2)$$

then reduce to

$$\nabla \cdot \vec{u} = 0$$

and

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}(\nabla p + \eta \nabla^2 \vec{u})$$

Incompressible fluids

- In order to solve these equations simplifications have to be made – first one is to assume that the fluid is **incompressible**
- This means that the density ρ is constant in space as well as in time
- Navier-Stokes equation (1)

$$\cancel{\frac{\partial \rho}{\partial t}} + \nabla \cdot (\rho \vec{u}) = 0$$

ρ constant in space

ρ constant in time

$$\rho \nabla \cdot \vec{u} = 0$$

$$\boxed{\nabla \cdot \vec{u} = 0}$$

Incompressible fluids

- In order to solve these equations simplifications have to be made – first one is to assume that the fluid is **incompressible**
- This means that the density ρ is constant in space as well as in time
- Navier-Stokes equation (2)

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho(\vec{u} \cdot \nabla)\vec{u} + \nabla p - \eta \nabla^2 \vec{u} - \cancel{(\lambda + \eta)\nabla(\nabla \cdot \vec{u})} = \vec{f}$$

$= 0$ because $\nabla \cdot \vec{u} = 0$ (previous eq)

$$\rho \left[\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u}\right] = -(\nabla p + \eta \nabla^2 \vec{u}) + \boxed{\vec{f}}$$

→ usually 0 or grouped with p

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}(\nabla p + \eta \nabla^2 \vec{u})$$

inertial term: $(\vec{u} \cdot \nabla)\vec{u}$

viscous term: $\nabla p + \eta \nabla^2 \vec{u}$

Incompressible fluids with no viscosity

- A further assumption is that the fluid has no viscosity ($\eta = 0$)

- Navier-Stokes equations are then reduced to

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}(\nabla p + \eta \nabla^2 \vec{u}) \longrightarrow \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}\nabla p$$

These are also called the incompressible inviscid Navier-Stokes equations

Incompressible fluids: what and when can we neglect?

- Let us step back and consider again the incompressible Navier-Stokes equations with viscosity:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\frac{1}{\rho}(\nabla p + \eta \nabla^2 \vec{u})$$

for a flow with some characteristic velocity U, in a spatial region with characteristic length ℓ.

- We can look at the effects of scale by introducing the following quantities:

$$\vec{u}^* = \frac{\vec{u}}{U} \qquad \vec{x}^* = \frac{\vec{x}}{\ell} \qquad \vec{y}^* = \frac{\vec{y}}{\ell} \qquad \vec{z}^* = \frac{\vec{z}}{\ell} \qquad t^* = \frac{Ut}{\ell} \qquad p^* = \frac{\ell}{\eta U} p$$

- Substituting these into Navier-Stokes equations results in:

$$\nabla \cdot \vec{u}^* = 0$$

$$\frac{\partial \vec{u}^*}{\partial t^*} + (\vec{u}^* \cdot \nabla)\vec{u}^* = \frac{1}{Re}[-\nabla p^* + \nabla^2 \vec{u}^*] \qquad \text{with} \qquad Re = \frac{\rho U \ell}{\eta} \qquad \text{Reynolds number}$$

Incompressible fluids: what and when can we neglect?

- At this point, we can drop the stars and write:

$$\nabla \cdot \vec{u} = 0$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = \frac{1}{Re}[-\nabla p + \nabla^2 \vec{u}]$$

- Re is a dimensionless parameter called the Reynolds number and is given by:

$$Re = \frac{\rho U \ell}{\eta}$$

For water η =10^{-3} Pa·s at 20 °C; 0.25 × 10^{-3} Pa·s at 100 °C

- This is why you can use scale models of ships and airplanes in tanks and wind tunnels

- Reynolds number sets the smallest scales of turbulent motion (at Re between 1000-10000)

- Different regimes of behavior of fluid flow are:

$Re \ll 1$	viscous effects dominate over inertial effects
$Re \approx 1$	viscous effects comparable to inertial effects
$Re \gg 1$	inertial effects dominate over viscous effects

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = \frac{1}{Re}[-\nabla p + \nabla^2 \vec{u}]$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = \frac{1}{Re}[-\nabla p + \nabla^2 \vec{u}]$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = \frac{1}{Re}[-\nabla p + \nabla^2 \vec{u}]$$

Reynolds number

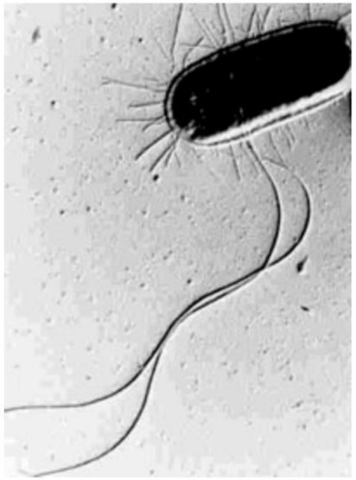
- Reynolds number

$$Re = \frac{\rho U \ell}{\eta}$$

- Can also be thought of as a ratio of inertial to viscous forces ($F = ma$ vs. $F = 6\pi\eta l v$)

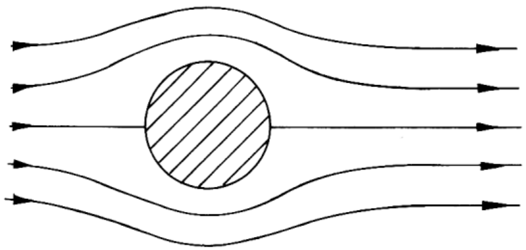
Michael Phelps
$Re \approx 10^4$

Guppy Fish
$Re \approx 10^2$

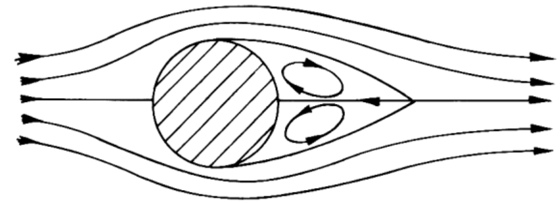


E. Coli
$Re \approx 10^{-4}$

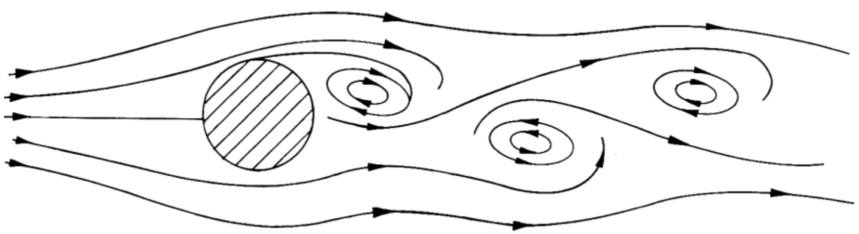
Reynolds number – fluid velocity streamlines



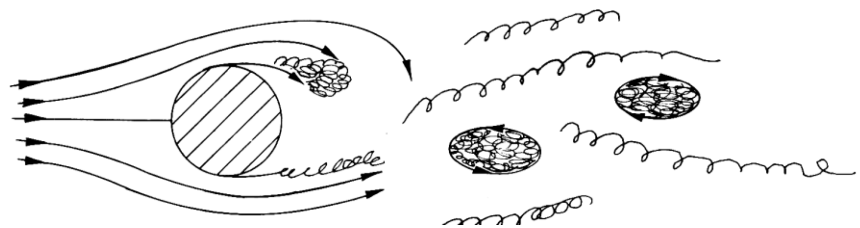
$Re \approx 10^{-2}$



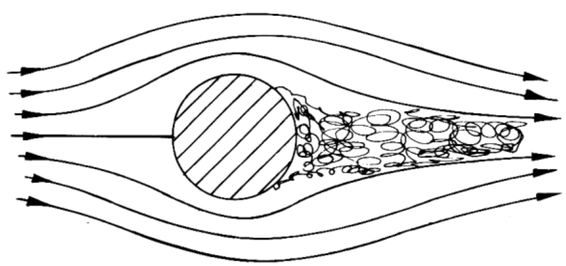
$Re \approx 20$



$Re \approx 100$



$Re \approx 10^{4}$



$Re \approx 10^{6}$

Incompressible fluids

- Some typical values:

  sperm cells ≈ 10^{-2}

  blood flow in brain ≈ 10^{2}

  blood flow in aorta ≈ 10^{3}

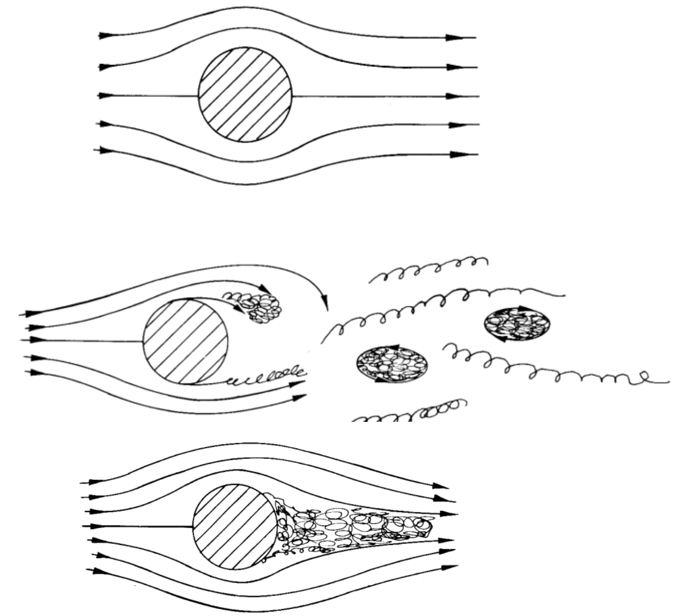
  ---

  onset of turbulent flow

  swimming ≈ 10^{4}

  blue whale ≈ 3×10^{8}

  large ship ≈ 5×10^{9}



- In the case of microfluidics, the incompressible Navier-Stokes equations are often replaced with their low Reynolds number limits

- To do this, we take the limit $Re \to 0$ of

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u} = \frac{1}{Re}\left[-\nabla p + \nabla^2\vec{u}\right]$$

which results in equations:

$$Re\cdot\left[\frac{\partial \vec{u}}{\partial t} + (\vec{u}\cdot\nabla)\vec{u}\right] = -\nabla p + \nabla^2\vec{u}$$

$Re \to 0$

so $\nabla p = \nabla^2\vec{u}$

$$\nabla\cdot\vec{u} = 0$$

$$\nabla p = \nabla^2\vec{u}$$

Incompressible fluids – boundary conditions

- As with all other continuum theories, Navier Stokes equations require boundary and initial conditions
- For initial conditions, we have to specify the velocity and pressure field in the fluid at time $t = 0$
- For the **solid – fluid interface** we have two types of boundary conditions:
  1. **No-penetration** – the fluid is not moving into the wall
     - this implies that the flow in directions normal to the wall must be zero at the wall:

$$\vec{u} \cdot \hat{n}\Big|_{\text{interface}} = 0$$

  2. **No-slip** – based on the experimental observation that at the interface between a fluid and solid, the fluid is not moving in the tangential direction. This is stated as:

$$\vec{u} \times \hat{n}\Big|_{\text{interface}} = 0$$

Taken together, these two can be simply written as: $\vec{u}\Big|_{\text{interface}} = 0$

Boundary conditions

- The no-slip boundary condition is based on experimental observation
- This condition does not always hold – fluids such as rarified gasses can slip along a solid surface
- To describe this behavior we will introduce a parameter called the Knudsen number Kn :

$$Kn = \frac{\lambda}{\ell}$$

where λ is the molecular mean free path and ℓ is a characteristic length of the system under consideration

- The molecular mean free path is the average distance traveled by a molecule between collisions
- In rarified gases the mean free path becomes large, comparable to the system size, so Knudsen number approaches 1
- In micro and nanofluidics, the **Knudsen number** becomes large because the **system size decreases** and becomes **comparable with the mean free path**

Boundary conditions

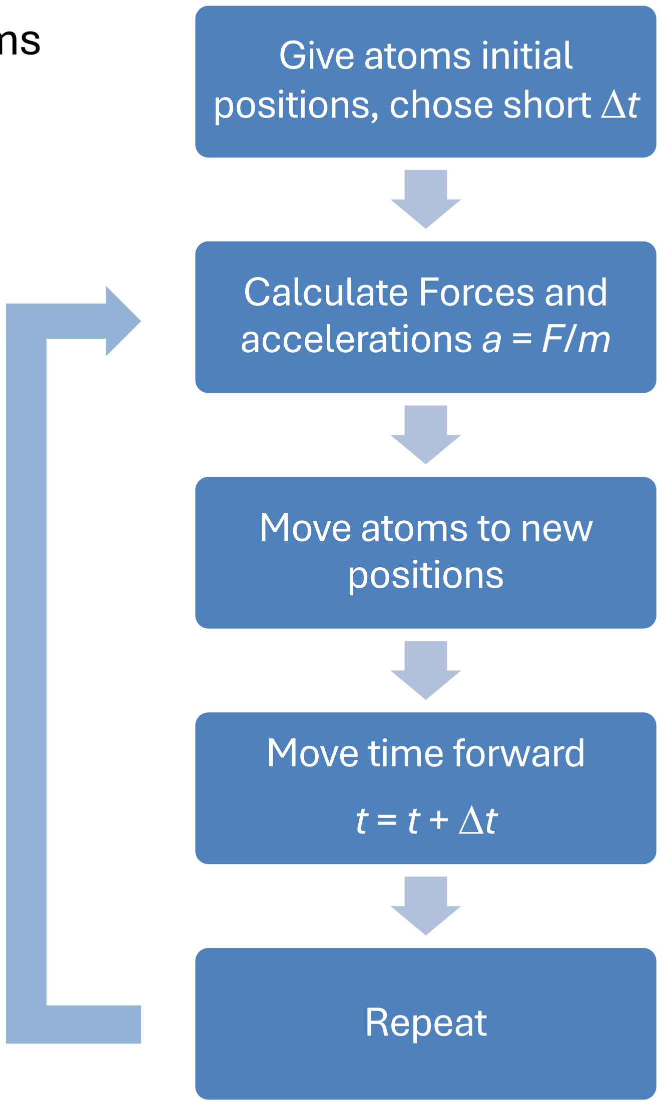
- As a rule of thumb, we can distinguish between the following regimes of fluid dynamics:

$Kn < 10^{-4}$	Navier-Stokes equations, no-slip boundary condition applies
$10^{-4} < Kn < 0.1$	Navier-Stokes equations, no-slip boundary does not apply
$0.1 < Kn < 10$	transitional flow regime
$Kn > 10$	free molecular flow

- In the cases when Navier-Stokes equations no longer hold ($Kn > 0.1$) we must use approaches based on molecular dynamics

Molecular dynamics

- Molecular dynamics is a type of computer simulation based on calculating the interaction force between atoms and molecules
- The technique begins with a collection of molecules distributed in space, with each molecule having a random velocity
- Velocities are then integrated forward in time to arrive at new molecular positions
- Intermolecular forces at the new time step are computed again and used to evolve the system forward in time



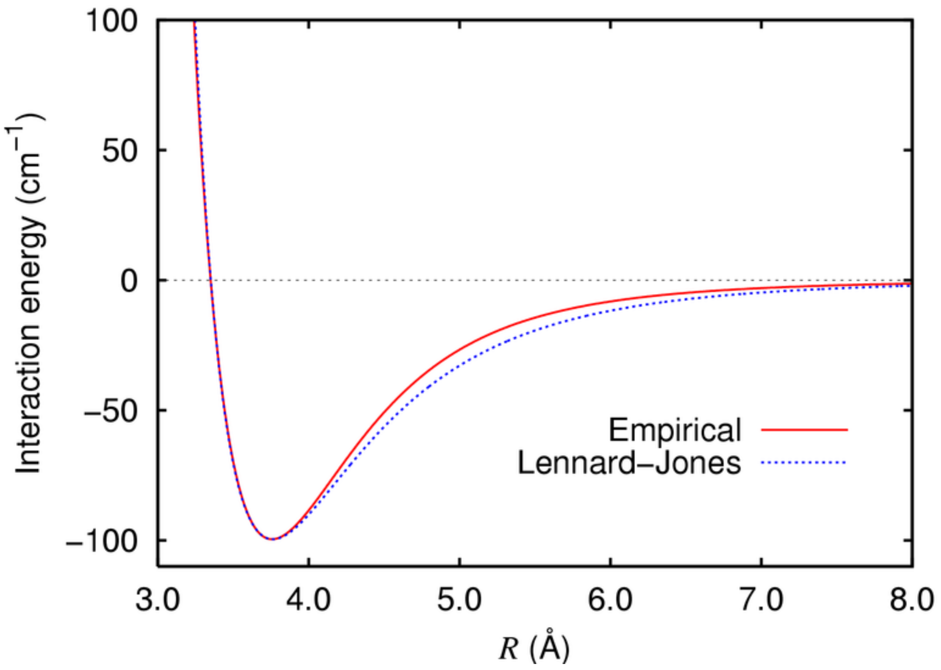
Molecular dynamics

- Interaction between molecules is usually described using the Lennard – Jones potential

$$V_{ij}(r) = 4\varepsilon\left[c_{ij}\left(\frac{\sigma}{r}\right)^{12} - d\left(\frac{\sigma}{r}\right)^{6}\right]$$

repulsive

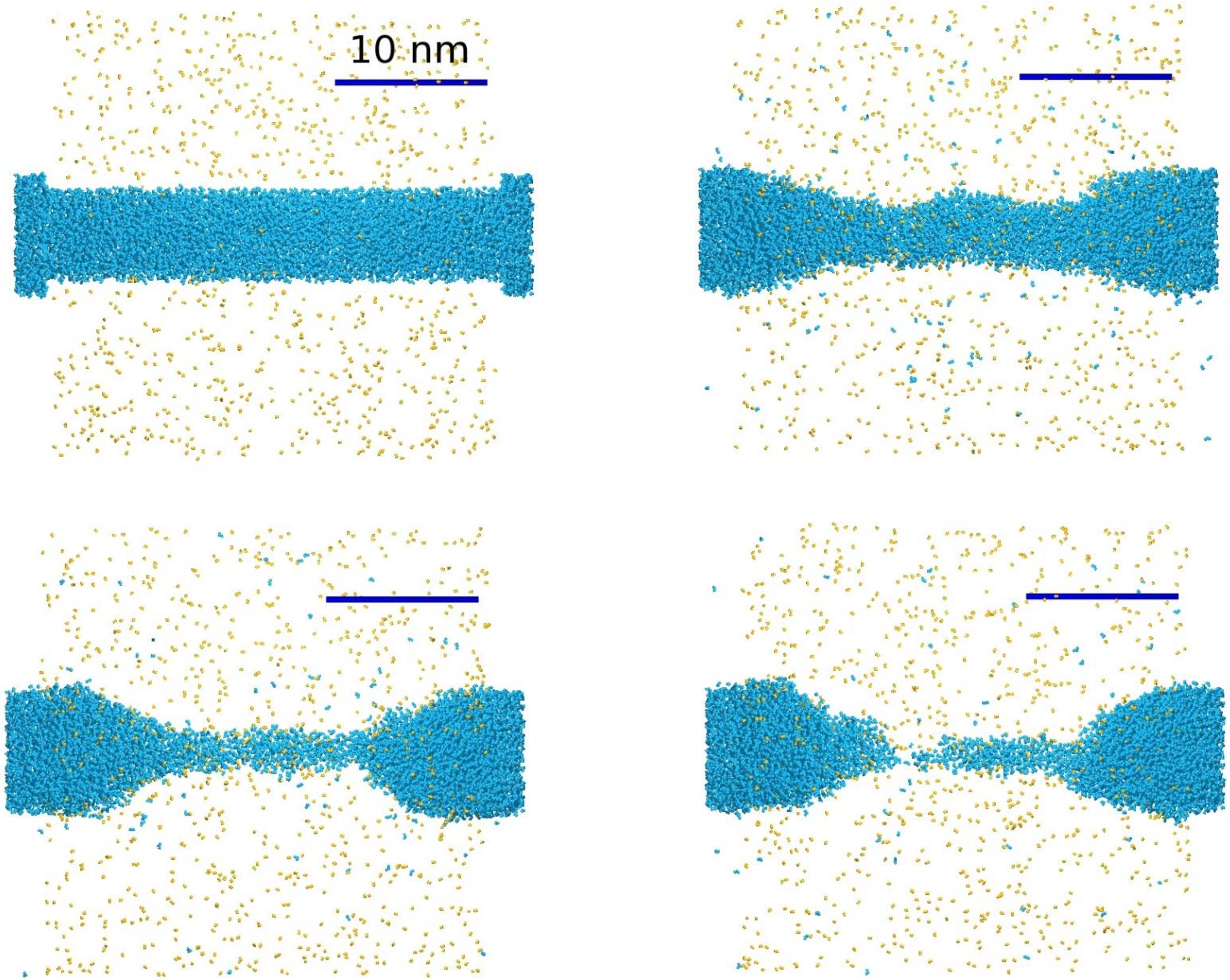
attractive



Interaction energy for two argon atoms

Molecular dynamics

- Breakup of a propane jet in nitrogen gas



Physical models for micro and nanosystems

Chapter 7: Microsystems for biology

Part 2: Heat Transport

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Heat transfer

- Heat can be transferred from one place or body to another in three ways:
  - Conduction
  - Convection
  - Radiation

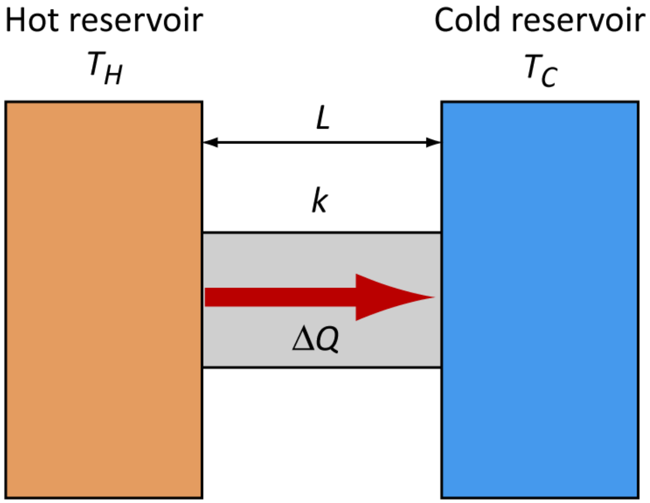
- These result in different boundary conditions

Heat transfer - conduction

- Consider an object with a cross-sectional area A and thickness L, with faces maintained at temperatures T_H and T_C

- ΔQ is the energy (heat) transferred through the slab, from the hot face to the cold face in time Δt

- The conduction rate P_{cond} is:



$$P_{cond} = \frac{\Delta Q}{\Delta t} = kA\frac{T_H - T_C}{L}$$

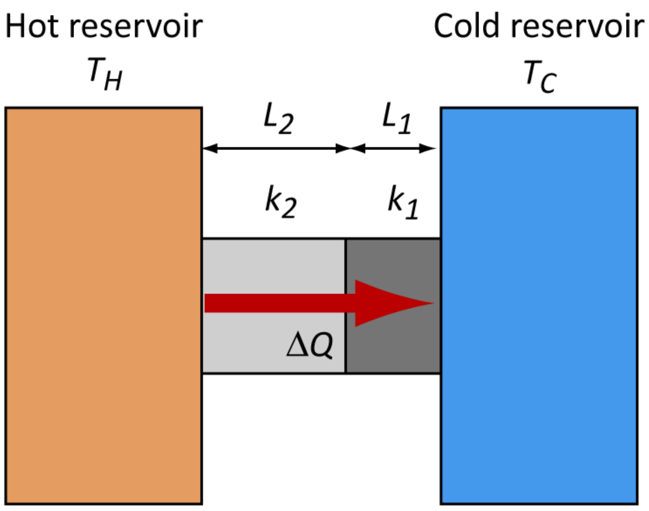
where k is the thermal conductivity, specific to a given material

Thermal conductivities

Substance	Thermal conductivity k ($Jm^{-1}s^{-1}\,°C^{-1}$) at room temperature
Silver	420
Copper	380
Aluminum	200
Steel	40
Glass	0.84
Water	0.56
Wood	0.1
Air	0.023

Heat transfer - conduction

- We will now consider heat conduction through an object composed of two materials, with different thicknesses and different thermal conductivities



- If the temperatures do not change with time, the conduction rates through the two materials must be equal

- If T_X is the temperature of the interface between the two materials, we can write:

$$P_{cond} = \frac{k_2 A(T_H - T_X)}{L_2} = \frac{k_1 A(T_X - T_C)}{L_1}$$

- After a bit of algebra we get:

$$P_{cond} = \frac{A(T_H - T_C)}{\frac{L_1}{k_1} + \frac{L_2}{k_2}}$$

$\frac{L}{k}$ is additive

For a single thermal link we had:

$$P_{cond} = kA\frac{T_H - T_C}{L}$$

Heat transfer - conduction

- In the general case, we divide the object into infinitesimally small segments. We can then rewrite the equation

$$P_{cond} = \frac{\Delta Q}{\Delta t} = kA\frac{T_H - T_C}{L}$$

in the following form (Fourier's law):

$$\frac{1}{A}\frac{dQ}{dt} = \vec{q} = -k\nabla T$$ where $\vec{q}$ is the heat flux through surface with area A

- For a given object with volume V, the conservation of energy says that the flux of energy (heat) through the surface is equal to the change in thermal energy.
- Thermal energy density is given by $\rho c_p T$ where ρ denotes the density of the material, c_p its specific heat and T its temperature
- Change in energy is equal to:

$$\frac{dE}{dt} = \frac{dQ}{dt} = \frac{d}{dt}\int \rho c_p T dV$$

Heat transfer - conduction

- Change in energy is equal to:

$$\frac{dQ}{dt} = \frac{d}{dt}\int \rho c_p T dV$$

- From energy conservation, this change in energy is equal to the integrated heat flux:

$$\frac{dQ}{dt} = \frac{d}{dt}\int \rho c_p T dV = -\int_S \vec{q}\cdot\hat{n}dS$$

We can use Gauss' theorem on the right hand side:

$$\int_S \vec{q}\cdot\hat{n}dS = \int_V (\nabla\cdot\vec{q})dV$$

This way we get from the expression for energy conservation:

$$\frac{d}{dt}\int \rho c_p T dV = -\int_V (\nabla\cdot\vec{q})dV$$

as the volume is arbitrary, we get:

$$\int \left[\rho c_p \frac{\partial T}{\partial t} + \nabla\cdot\vec{q}\right] dV = 0$$

Heat transfer - conduction

- As the volume of integration is arbitrary, we finally get:

$$\rho c_p \frac{\partial T}{\partial t} + \nabla \cdot \vec{q} = 0$$

- By inserting Fourier's law $\vec{q} = -k\nabla T$, we get:

$$\rho c_p \frac{\partial T}{\partial t} = \nabla \cdot (k\nabla T)$$

This is valid for an anisotropic inhomogeneous medium where k could be a tensor (just like in elasticity) while $\rho,\ c_p,\ k$ are in general functions of position

- In a homogeneous and isotropic material we can simplify this equation to:

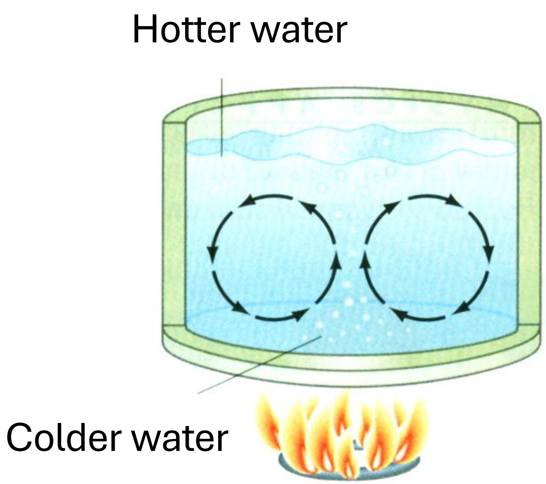
$$\boxed{\frac{\partial T}{\partial t} = \frac{k}{\rho c_p} \nabla^2 T}$$

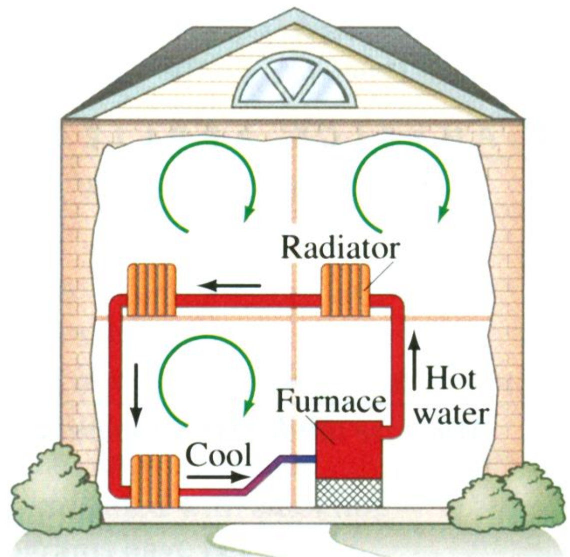
Heat transfer – convection

- **Convection –** energy transfer which occurs when a fluid (air, water etc.) comes in contact with an object whose temperature is higher than that of the fluid

  Heat is transported via the motion of the fluid

  This motion can be forced (forced convection) or natural (due to heat expansion of the fluid)





Convective flows in a house

Heat transfer – radiation

- **Radiation –** heat exchange via electromagnetic waves, no medium is required
- The rate P_{rad} at which an object emits energy via electromagnetic radiation depends on the object's surface area A and the temperature T of that area (in Kelvins) and is given by:

$$P_{rad} = \sigma \varepsilon A T^4$$

where $\sigma = 5.67 \cdot 10^{-8} W/m^2K^4$ is the Stefan-Boltzmann constant

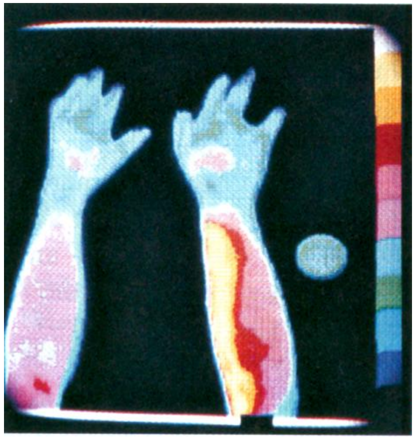
ε is the emissivity of the object and has a value between 0 (shiny) and 1 (black matte)

- The rate P_{abs} at which an object absorbs energy via thermal radiation from its environment is:

$$P_{abs} = \sigma \varepsilon A T_{env}^4$$

- Because an object can radiate energy to the environment and also absorb it, the net rate of energy exchange due to radiation (from the point of view of the object) is going to be:

$$P_{net} = P_{abs} - P_{rad} = \sigma \varepsilon A (T_{env}^4 - T^4)$$



Thermal scan of hands

Heat transfer – boundary conditions

- In order to complete the description of problems involving heat transfer, we must specify initial and boundary conditions
- For initial conditions, we must simply specify the temperature of the body at every point in space at time $t = 0$

$$T(x, y, z, 0) = f(x, y, z)$$

1. **Controlled temperature**

   This boundary condition (Dirichlet type) applies when a device has zones with a constant temperature or if a large solid mass is in contact with the device. This large mass is then supposed to remain at a constant temperature

$$T\Big|_S = T_A$$

2. **Symmetry plane**

   This boundary condition (von Neumann type) applies when the device geometry has one or more planes of symmetry and when the heat sources are implemented with the same symmetry i.e. no heat passes through these places. We can then impose on the symmetry planes:

$$k\nabla T \cdot \hat{n}\Big|_S = k\frac{\partial T}{\partial n} = 0$$

Heat transfer – boundary conditions

3. **Insulated wall**

   This boundary condition (von Neumann type) applies for materials with low thermal conductivity. The heat flow through such materials is negligible. We then have:

$$k\nabla T \cdot \hat{n}\Big|_S = k\frac{\partial T}{\partial n} = 0$$

4. **Heat transfer by convection**

   Here, the body is in contact with a fluid occupying a large volume. This could be a huge liquid reservoir or the atmosphere. This fluid is assumed to have a temperature T_A. The corresponding boundary condition is then:

$$k\frac{\partial T}{\partial n}\Big|_S = -h(T - T_A)\Big|_S$$

where h is the coefficient of convective exchange of the boundary. This value depends on the nature of the fluid in the vicinity of the wall

Heat transfer – boundary conditions

5. **Heat transfer by radiation**

   Objects emit electromagnetic radiation which is absorbed by surrounding objects.

   This boundary condition is expressed using:

$$k \frac{\partial T}{\partial n}\bigg|_S = -\varepsilon\sigma(T^4 - T_A^4)\Big|_S$$

Which mechanism is most important?

If the heat is carried away by a:	**then (on the microscale), we need to consider**
solid	conduction
fluid (liquid or gas)	convection
there is no medium (vacuum) or the temperature difference is large	radiation